Ben Johnston's Extended Just Intonation: A Guide for Interpreters

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I: Theoretical Foundation

All of Ben Johnston's music using extended just intonation is built on a rational notation system which accurately and elegantly describes intervallic and harmonic relationships. The system, however, is not easy to grasp for someone not completely familiar with the complexities of just intonation. Therefore, Part I of this article will explain the foundations of Ben Johnston's just intonation and his notation system. Part II will present music examples which illustrate certain types of harmonic and scale structures. This article is directed at interpreters but should be of value to theorists and composers interested in just intonation and Johnston's music.

Extended just intonation is a tuning system based on the "pure" intervals of the overtone series: intervals generated therefore from whole-number ratios. This is in contrast to temperaments, both equal and unequal, which use compromised intervals, often based on irrational numbers. (Conventional equal temperament, for example, divides the octave into twelve equal semitones of a size expressed as the ratio of 1 to the twelfth root of 2, or 1:1.05946, approximately.)

> Just intonation is simply the easiest way to tune musical intervals by ear. It results in greatly heightened purity and clarity of sound for two reasons: it eliminates acoustic beats to the maximum possible, and second, it exploits resonance by utilizing harmonically simple combinations of pitches. The term extended refers to the use of higher overtones than the first six partials.[1]

Extended just intonation also allows for modulation to a potentially infinite number of pitch centers. The system can be closed either in an arbitrary way or can close itself depending on the compositional choices made in regard to harmonic regions. For instance, Johnston presented a closed system of fifty-three notes per octave in his article "Scalar Order as a Compositional Resource" (Johnston 1964). He used this system in his String Quartet Number 2. Although the system is arbitrarily closed, it has an inherent symmetrical beauty with a large but limited range of modulation. The system is also based on the 2,3,5 prime number limit—that is, no ratios involving prime factors larger than 5 are employed.

The theory of limits refers to the prime-number partials which are being used to generate intervallic relationships. The 2,3,5 limit also implies any multiple of those three numbers. The equal-tempered twelve-note scale presumes to represent the five limit, although it more aptly approximates the Pythagorean 2,3 limit scale. Much of Johnston's music, however, uses the prime numbers 7, 11, 13, and higher, which cannot be approximated in twelve-note equal temperament. In Johnston 1977, he presents the prime number 7 in regard to lattice structures and microtonal scale formations.

Compositions by Johnston that use the 2,3,5 limit include *Five Fragments* (1960) for alto, oboe,'cello, and bassoon; *A Sea Dirge* (1960) for soprano, flute, oboe, and violin; String Quartet Number 2 (1964), Sonata for Microtonal Piano/*Grindelmusic* (1964); and String Quartet Number 3 (1966).

Works that use seventh-partial harmonic structures include *One Man* (1967) for trombone; *Rose* (1971); Mass (1972); and String Quartet Number 4 (1973). *Innocence* (1975) and *Two Sonnets of Shakespeare* (1979) use 2, 3, and 11 only. *Diversion* (1979) for eleven instruments, String Quartet Number 6 (1980), and Toccata (1984) for 'cello use the seventh partial and eleventh partial. Works using the thirteenth include Duo for Two Violins (1978), String Quartet Number 5 (1979), *Sonnets of Desolation* (1980), String Quartet Number 7 (1984), and String Quartet Number 8 (1986).

Two other works use higher partials: Suite for Microtonal Piano (1978) and *Twelve Partials* (1980) for microtonal piano and flute. However, only the higher partials of C are used in both works. String Quartet Number 9 (1987) uses all the prime-number partials up to and including the thirty-first.[2]

All of Johnston's extended-just-intonation notation is built on C as 1/1. All C's are multiples of the prime identity. This does not mean that his music is always in C or that C is the tuning note.[3] From this 1/1 Johnston builds a major chord 1/1: 5/4: 3/2, forming the harmonic relationship of 4:5:6 (C–E–G). This triad, like all just major triads, is beatless and represents a simple musical perfection found in the fourth, fifth, and sixth partials of the harmonic series. The interval between the 5/4 and the 3/2, or between the third and fifth of any 4:5:6 triad, is the just minor third, or the 6/5.

This triadic process is repeated on the 3/2 and its inversion the 4/3 (G and F, respectively). This diatonic scale collection provides the "natural" notes or reference notes upon which all the accidentals function and to which they refer as shown in Example 1.

1/1:	5/4:	3/2	=	C	E	G
3/2:	15/8:	9/8	=	G	B	D
4/3:	5/3:	1/1	=	F	A	C

EXAMPLE 1

This scale is comprised of two similar but not identical tetrachords, separated by a 9/8 whole tone, as shown in Example 2.

cent value:	0		204		386		498		702		814		1088		1200
ratio to C:	1/1		9/8		5/4		4/3		3/2		5/3		15/8		1/1
notation:	C		D		E		F		G		A		B		C
difference*:		9/8		10/9		16/15		—9/8—		10/9		9/8		16/15	

*ratio of adjacent pitches

EXAMPLE 2

One can then continue to build a secondary group of triads (4:5:6) on the 9/8, 5/4, 5/3, and 15/8, as presented in Example 3. From this one derives a new "natural" note (the 27/16 A+) and four "sharp" notes. The flats are obtained by different means and will be explained later.

At this point one must examine the relationships between the new notes and their diatonic progenitors. The triad built on the 5/3 (A) produces C#

25/24. From this it can be assumed that the 25/24 ratio is the sharp (#) ratio and can be applied to any diatonic note to produce the sharpened form of that note. This raises a note by approximately 70.6 cents. Example 4 shows this process.

ratio to C:	9/8	45/32	27/16
notation:	D	F#+	A+
	5/4	25/16	15/8
	E	G#	B
	5/3	25/24	5/4
	A	C#	E
	15/8	75/64	45/32
	B	D#	F#+

EXAMPLE 3

D	9/8 x 25/24	=	75/64	D#
G	3/2 x 25/24	=	25/16	G#
F	4/3 x 25/24	=	25/18	F#

EXAMPLE 4

Notice that this F# does not correspond to the F#+ occurring in the triads built on 9/8 and 15/8. The questions arise: What is the relationship between these F#s, and where does the 25/18 belong harmonically? The answer to the first question is easily solved (keeping in mind that ***subtraction of intervals*** is equivalent to ***dividing the ratios*** of the two pitches involved):

45/32 ÷ 25/18 = 81/80, the syntonic comma (21.5 cents)

The 45/32 is therefore 81/80 higher than the 25/18. Johnston uses a + to indicate this tuning correction. Therefore, 25/18 is called F# and 45/32 becomes F#+. The 25/18 is the 5/4 in a triad built on 10/9. By combining the (#) and the (+) Johnston has expressed the tuning correction variously called by theorists throughout history (there is still no consensus on the name of this interval) as "larger limma," "larger chromatic semitone," or "major chroma," which has the ratio of 135/128 and raises a note 92 cents.

The – is used to indicate a note 81/80 below another. A perfect fifth below A would be D– (10/9) and is an 81/80 lower than 9/8. Likewise, a perfect fifth up from D is A+ (27/16), an 81/80 higher than the 5/3. The use of + and – signs is crucial in the understanding of extended just intonation. Among the many applications, one of the most problematic is found on harmonic structures built on D and B. To illustrate the problem in simple terms, a major triad tuned above B would be B, D#, and F#+ and a major chord from D would be D, F#+, and A+. Likewise a major triad built up from B♭– would be B♭– D– and F.

To obtain the flats in this just scale, one should understand the relationship between major and minor triads. This relationship may be expressed by two terms coined by Harry Partch: "otonal" and "utonal" (shortenings of "overtone-al" and "undertone-al"). Johnston continues to use these terms. In this sense the minor triad is a mirror inversion of the major chord. This inversion can be explained as a subharmonic relationship or subharmonic series descending from the 1/1. For example, as explained above, 1/1, 5/4, 3/2 is C, E, and G—a major triad. By inverting this process to descend a 5/4 and 3/2 from 1/1, a sequence of 1/1, 8/5, and 4/3 results. This forms a minor triad: C, A♭, and F. Example 5 shows this procedure within the five limit. Whereas Partch stopped at the eleventh partial, Johnston is currently using numbers up to thirty-one. These higher partials will be presented later.

F		A♭		C		E		G
4/3		8/5		1/1		5/4		3/2
	6/5		5/4		5/4		6/5	

EXAMPLE 5

As one can readily see 8/5 is the inversion of 5/4 and 4/3 is the inversion of 3/2. Any ratio which has the larger prime factor (a prime number or any whole-number multiple of it) in its numerator is considered an otonal ratio. Conversely, a utonal ratio always has the largest prime factor in its denominator. This fact, however, does not address the harmonic function of a note. This is entirely dependent on usage and context. Although the utonal triad C–A♭–F is generated from C, the harmonic function can be viewed from the tonal tradition which dictates that the F is the root. Some of Johnston's music accepts this traditional tonal orientation, but much of it does not. Otonal and utonal harmonic structures will be further explained below. One can continue this process from the primary 3/2 and 4/3 to obtain other flats. Example 6 shows the utonal triads formed on the 3/2 and 4/3.

Notice that the D♭ – 16/15 occurs as a just "semitone" and is the inversion of the 15/8 (B). The D♭ is obtained by subtracting a 25/24 from the 9/8 (that is, dividing the latter ratio by the former) which yields 27/25. The

3/2	6/5	1/1
G	E♭	C
4/3	16/15	16/9
F	D♭–	B♭–

EXAMPLE 6

27/25 occurs in a minor triad built on the 9/5 (B♭): 9/5, 27/25, 27/20 or B♭, D♭, and F+.

One can apply an alternative procedure to obtain the flats. Example 7 illustrates that by simply inverting the ratios corresponding to the diatonic naturals, the flats are generated.

9/8:	16/9	B♭–
5/4:	8/5	A♭
5/3:	6/5	E♭
15/8:	16/15	D♭–

EXAMPLE 7

With the exception of the double sharps and flats, we now have all the tuning accidentals of the 2,3,5 limit. These can be combined in numerous ways, as will be demonstrated in Part II below.

A just enharmonic scale is shown in Example 8, with Johnston's notation, and the ratio and cent values from C (1/1, 0 cents). The ratios of the diatonic degrees are marked with a following colon. This twenty-two-note scale is unequal and is composed of adjacent intervals of 25/24 (70.6 cents), and three intervals traditionally called by their Greek names: the greater diësis 648/625 (62.6 cents), the diësis 128/125 (41 cents), and the small diësis 3125/3072 (28 cents).

Example 8 is not a useful scale for harmonic usage. A more useful just scale collection in regard to harmonic potential is presented in Example 9. This collection provides major and minor triads on, and dominant triads to, all the basic just scale degrees: 1/1, 16/15, 9/8, 6/5, 5/4, 4/3, 3/2, 8/5, 5/3, 16/9, and 15/8. It is a twenty-five-note scale.

One can readily obtain other sharps and flats by constructing major and minor triads on the secondary just notes, and then on triads from the results of those triads. This process can be continued infinitely and will soon involve numerous combinations of accidentals, as is implied by the pitch lattice

note	ratio	cents
C	1/1:	0
C#	25/24	70
D♭	27/25	133
D	9/8:	204
D#	75/64	275
E♭	6/5	316
E	5/4:	386
F♭	32/25	428
E#	125/96	456
F	4/3:	498
F#	25/18	568
G♭	36/25	632
G	3/2:	702
G#	25/16	773
A♭	8/5	814
A	5/3:	884
A#	125/72	954
B♭	9/5	1018
B	15/8:	1088
C♭	48/25	1130
B#	125/64	1158
C	1/1	0

EXAMPLE 8: THE JUST ENHARMONIC SCALE

presented in Johnston 1977 and in the "Introduction" to the score of String Quartet Number 6. The 2,3,5 lattice (Example 10) is extremely useful for understanding triadic and harmonic relationships and the potentials of extended just intonation. The cent values are given in the lowest part of the box and the notation is given in the upper right of the box.

From this lattice one can project 3/2 relationships vertically and 5/4 relationships horizontally. From 1/1 (at the center of the lattice) up to 3/2 is a perfect fifth and from 1/1 down to 4/3 is a descending perfect fifth (utonal). To the right of 1/1 is 5/4 a major third up and to the left an 8/5 a major third down. Also, going diagonally up and to the right of any ratio one can find the leading tone (15/8 is the leading tone to 1/1). The minor third is found by going diagonally up and to the left. Major triads are formed from any ratio by going to the ratios immediately to the right and also immediately above. Minor triads are formed by going immediately up and immediately to the left.

note	ratio	cents
C	1/1:	0
C#	25/24	70
C#+	135/128	92
D♭–	16/15	112
D–	10/9	180
D	9/8:	204
D#	75/64	275
E♭	6/5	316
E	5/4	386
E+	81/64	408
F♭	32/25	428
F	4/3:	498
F+	27/20	520
F#+	45/32	590
G♭	36/25	632
G	3/2:	702
G#	25/16	772
A♭	8/5	814
A	5/3:	884
A+	27/16	906
A#+	225/128	976
B♭–	16/9	996
B♭	9/5	1018
B	15/8:	1088
C♭	48/25	1130
C	1/1	0

EXAMPLE 9: THE JUST ENHARMONIC SCALE

As mentioned above, Johnston has also used the prime numbers of 7, 11, 13, 17, 19, and higher in his music. Each one of these ***partial chroma*** requires a notational symbol. These are given below up to and including the thirty-first partial, which is the current limit used in Johnston's music.

The "septimal comma," or the amount the minor seventh (9/5) exceeds the seventh harmonic (7/4), is indicated by a 7. This interval has the ratio 36/35 and when applied to a note lowers it by 36/35 (49 cents). The cent value for 9/5 is 1018. By lowering the 9/5 by 49 cents one arrives at 969 or the cent value of 7/4. The utonal interval is notated by an inverted numeral seven, ㄥ, and raises a note by 36/35. Thus the seventh partial (7/4) of C is notated B7. The corresponding utonal interval, 8/7, is notated as Dㄥ–.

C♭+ 243 125 1151	E♭+ 243 200 337	G+ 243 160 723.4	B+ 243 128 1109.7	D#+ 1215 1024 296	Fx++ 6075 4096 682.4	Ax++ 30375 16384 1068.7
F♭+ 162 125 449	A♭+ 81 50 835	C+ 81 80 21.5	E+ 81 64 408	G#+ 405 256 794	B#+ 2025 1024 1180.4	Dx+ 10125 4096 336.7
B♭♭ 216 125 947	D♭ 27 25 133	F+ 27 20 519.5	A+ 27 16 906	C#+ 135 128 92	E#+ 675 512 478.4	Gx+ 3375 2048 865
E♭♭ 144 125 245	G♭ 36 25 631	B♭ 9 5 1017.5	D 9 8 204	F#+ 45 32 590	A#+ 225 128 976.5	Cx+ 1125 1024 163
A♭♭ 192 125 743	C♭ 48 25 1129.3	E♭ 6 5 315.6	G 3 2 702	B 15 8 1088	D# 75 64 274.5	Fx+ 375 256 661
D♭♭- 128 125 41	F♭ 32 25 427.3	A♭ 8 5 813.6	C 1 1 0	E 5 4 386.3	G# 25 16 772.6	B# 125 64 1159
G♭♭- 512 375 539	B♭♭- 128 75 925.4	D♭- 16 15 111.7	F 4 3 498	A 5 3 884.3	C# 25 24 70.6	E# 125 96 457
C♭♭- 2048 1125 1037	E♭♭- 256 225 223.4	G♭- 64 45 609.7	B♭- 16 9 996	D- 10 9 182.4	F# 25 18 568.7	A# 125 72 955
F♭♭- 4096 3375 335	A♭♭-- 1024 675 721.5	C♭- 256 135 1108	E♭- 32 27 294	G- 40 27 680.4	B- 50 27 1066.7	D#- 125 108 253
B♭♭♭-- 16384 10125 833	D♭♭-- 2048 2025 19.5	F♭- 512 405 406	A♭- 128 81 792	C- 160 81 1178.4	E- 100 81 365	G#- 125 81 751
E♭♭♭-- 32768 30375 131	G♭♭-- 8192 6075 517.5	B♭♭- 2048 1215 904	D♭-- 256 243 90	F- 320 243 476.5	A- 400 243 863	C#- 250 243 49

EXAMPLE 10: 2,3,5 PITCH LATTICE—NOTATION, RATIO, AND CENT VALUES

The eleventh partial is represented by an upward-pointing arrow, ↑, for the otonal inflection (11/8) and a downward arrow, ↓, for the utonal one (16/11). This sign indicates the raising or lowering of a note by the interval of 33/32 (53 cents). To raise the 4/3 (cent value 498) by 11/8, or 53 cents, one arrives at 551, the cent value of 11/8 notated F↑.

The thirteenth partial is notated ^{13} and $^{\text{ƐI}}$, and raises or lowers a note by 26.8 cents, respectively. The ratio for this interval is 65/64 and is the amount the 13/8 (A^{13}♭) exceeds the 8/5.

The seventeenth partial is notated ^{17} and $^{\text{ㄥI}}$. These raise and lower a note by 34 cents, respectively. The ratio for this interval is 51/50 and is the amount the 17/16 (C^{17}♯) exceeds the 25/24.

The nineteenth partial is notated by ^{19} and $^{\text{6I}}$, which lower and raise a note by 18 cents, respectively. The ratio for this interval is 96/95 and is the amount the 6/5 exceeds the 19/16 (E^{19}♭).

The first work of Johnston to use prime-number partials above the nineteenth is String Quartet Number 9. In addition to higher-partial chroma generated from 7, 11 and 13, this quartet uses twenty-third- , twenty-ninth- , and thirty-first-partial chroma. For the notation of the twenty-third partial a ^{23} and a $^{\text{ƐZ}}$ are used to raise and lower a note by a 46/45 (38 cents). The 46/45 is the amount the 23/16 exceeds the 45/32 (F^{23}♯+). For the twenty-ninth partial a ^{29} and $^{\text{6Z}}$ are used to raise and lower a note by a 145/144 (about 12 cents). The 145/144 is the amount the 29/16 exceeds the 9/5. The thirty-first partial is represented by ^{31} and $^{\text{IƐ}}$. This is 31/30 (about 57 cents), which is the amount the 31/16 exceeds the 15/8.

Thus, the notation of the otonal harmonic structure using the prime-number partials up to thirty-one from 1/1 is given in Example 11. The utonal harmonic structure using the inversions of those same ratios from 1/1 is shown in Example 12.

1	3	5	7	11	13	17	19	23	29	31
1	2	4	4	8	8	16	16	16	16	16
C	G	E	B7	F↑	A^{13}♭	C^{17}♯	E^{19}♭	F^{23}♯+	B^{29}♭	B^{31}

EXAMPLE 11

1	4	8	8	16	16	32	32	32	32	32
1	3	5	7	11	13	17	19	23	29	31
C	F	A♭	Dㄥ−	G↓	E$^{\text{ƐI}}$♭	C$^{\text{ㄥI}}$♭	A$^{\text{6I}}$	G$^{\text{ƐZ}}$♭−	D$^{\text{6Z}}$−	D$^{\text{IƐ}}$♭−

EXAMPLE 12

Each one of these higher partials requires both an otonal and a utonal 2,3,5 lattice. Obviously, with the inclusion of higher partials and their extended relationships, more dimensions are needed for the lattice. In Johnston 1977 the composer presented the concept of multi-dimensional lattice structures using the prime number 7. This article is of tremendous consequence in the elaboration of Johnston's system of extended just intonation and the formation of microtonal scales. This article also implies the use of higher partials, and therefore "*n*-dimensional" lattice structures which are very difficult to express graphically.

Example 13 demonstrates the logical extension of the ideas presented in Johnston 1977 by providing additional dimensions (prime numbers up to 19) to the 2,3,5 lattice. For the sake of brevity this paper will only present a 2,3,7,11,13,17, and 19 otonal lattice built on the center column of the 2,3,5 lattice. Each column could produce a similar structure. This abbreviated lattice should serve to inform the reader of the enormous pitch resources of extended just intonation.

One more theory is needed to bring the system to its present state of usage in Johnston's music. In his introduction to the score of String Quartet Number 6, the dimensions of the lattice are expanded to include the seventh and eleventh partials combined (Example 14); e.g., seventh- and eleventh-partial relationships are generated *from* seventh- and eleventh-partial ratios. In order to render the lattice visually intelligible, the 5/4 columns of the 2,3,5 lattice have been separated. Thus, the 1/1, 5/4, and 8/5 each have a separate eight-dimensional otonal and utonal lattice. Although appearing overly complex at first, the lattice elegantly displays the simplicity of the harmonic relationships.

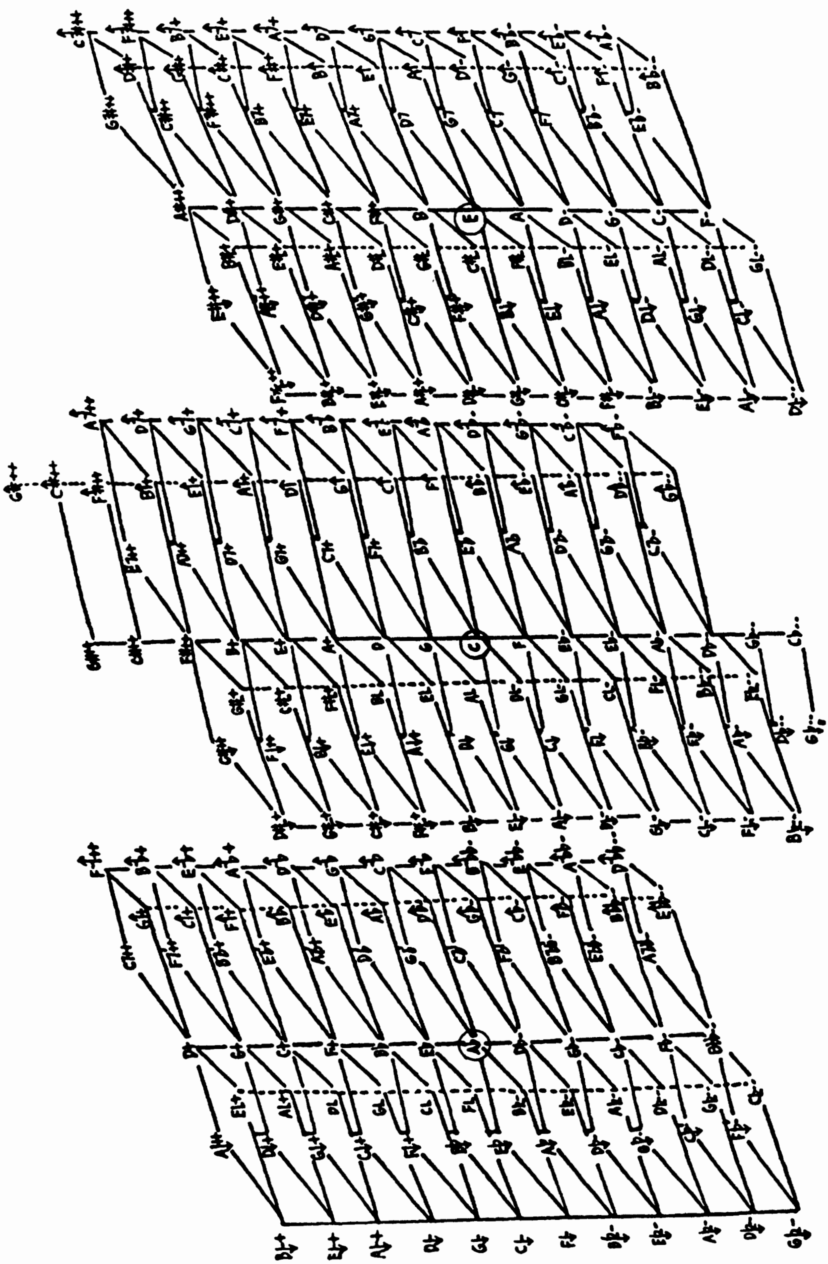
This lattice demonstrates and implies that otonal and utonal structures can be created on seventh, eleventh, thirteenth, and higher partials. Each of these combined partials requires additional dimensions in the complete lattice to express the necessary harmonic potential. Example 15 demonstrates how harmonic structures are generated from higher partials combined. The example presents a thirteen-limit otonal structure from C, a thirteen-limit otonal structure from 7/4 (extending upward from B7♭), a thirteen-limit utonal structure from 13/8 (extending downward from A^{13}♭) and a thirteen-limit otonal structure extending from the combined thirteen-otonal and eleven-utonal ratio, 13/11.

Since the lattices would be so numerous, only an abbreviated lattice (Example 16) for the 7/4 otonal is included in this article.[4] This lattice adds four more dimensions to the already abundant galaxy, and the utonal 7/4 lattice would add yet four more.

In the 7/4 otonal lattice the middle box (now the middle box on the furthest left column) is 7/4 (B7♭) and is functioning like 1/1 from which it is generated. In addition, the higher-partial combined ratios, such as 77/64,

B+ 243 128 1110	A7 ++ 1701 1024 878.6	E↑+ 2673 2048 461	G^{13}+ 3159 2048 750	B^{17}#+ 4131 4096 14.7	D^{19}+ 4617 4096 207
E+ 81 64 408	D7 + 567 512 176.6	A↑+ 891 512 959	C^{13}+ 1053 1024 48.3	E^{17}#+ 1377 1024 512.7	G^{19}+ 1539 1024 705.3
A+ 27 16 906	G7 + 189 128 674.7	D↑ 297 256 257	F^{13}+ 351 256 546.3	A^{17}#+ 459 256 1011	C^{19}+ 513 512 3.3
D 9 8 204	C7 + 63 32 1172.7	G↑ 99 64 755	B^{13}♭ 117 64 1044.4	D^{17}# 153 128 309	F^{19}+ 171 128 501.4
G 3 2 702	F7 + 21 16 470.7	C↑ 33 32 53	E^{13}♭ 39 32 342.4	G^{17}# 51 32 807	B^{19}♭ 57 32 999.4
C 1 1 0	Bㄥ 7 4 969	F↑ 11 8 551.3	A^{13}♭ 13 8 840.5	C^{17}# 17 16 105	E^{19}♭ 19 16 297.5
F 4 3 498	Eㄥ 7 6 267	B↓− 11 6 1049.3	D^{13}♭− 13 12 138.5	F^{17}# 17 12 603	A^{19}♭ 19 12 795.5
B♭− 16 9 996	Aㄥ 14 9 765	E↓− 11 9 347.4	G^{13}♭− 13 9 636.6	B^{17}− 17 9 1101	D^{19}♭− 19 18 93.6
E♭− 32 27 294	Dㄥ− 28 27 63	A↓− 44 27 845.4	C^{13}♭− 52 27 1134.6	E^{17}− 34 27 399	G^{19}♭− 38 27 591.6
A♭− 128 81 792	Gㄥ− 112 81 561	D↓− − 88 81 143.4	F^{13}♭− 104 81 432.7	A^{17}− 136 81 897	C^{19}♭− 152 81 1089.7
D♭− − 256 243 90	Cㄥ− 448 243 1059	G↓− − 352 243 641.5	B^{13}♭♭− − 416 243 930.7	D^{17}− − 272 243 195	F^{19}♭− 304 243 287.7

EXAMPLE 13: 2,3,7,11,13,17, AND 19 ABBREVIATED OTONAL PITCH LATTICE



EXAMPLE 14: 2,3,5,7,11,13 LATTICE FOR STRING QUARTET NO. 6

<table>
<tr><td></td><td></td><td></td><td>G¹³♭↑</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>E♭↑</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>A♭7 7 +</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>F7 +</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>D7</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>C</td><td>G</td><td>E</td><td>B♭7</td><td>F↑</td><td>A¹³♭</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>F¹³♭</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>D¹³♭ −</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>B¹³♭ ㄥ −</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>E¹³♭ ↑</td><td>G¹³↑</td><td>B¹³♭ ↑</td><td>D¹³♭7 ↑</td><td>A¹³♭↑ ↑</td><td>C¹³♭ ¹³↑</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>C</td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

EXAMPLE 15: HARMONIC STRUCTURES ON HIGHER PARTIALS COMBINED

C7 + 63 32 1172.7	E7 + 315 256 359	B♭7 7 + 441 256 941.5	F7 ↑+ 693 512 524	A^{13}♭7 819 512 813
F7 + 21 16 470.7	A7 + 105 64 857	E♭7 7 + 147 128 239.6	B♭↑ 231 128 1022	D^{13}♭7 273 256 111.3
1 B♭7 1 7 4 969	D7 35 32 155	A♭7 7 + 49 32 737.6	E♭↑ 77 64 320	G^{13}♭7 91 64 609.3
E♭7 7 6 267	G7 35 24 653	D♭7 7 49 48 35.6	A♭↑ 77 48 818	C^{13}♭7 91 48 1107.4
A♭7 14 9 765	C7 35 18 1151	G♭7 7 49 36 533.7	D♭↑ − 77 72 116	F^{13}♭7 91 72 405.4

EXAMPLE 16: 2,3,5,7,11, AND 13 ABBREVIATED OTONAL LATTICE ON 7/4

can function as either the seventh partial of 11/8 or the eleventh partial of 7/4. Likewise, the 91/64 can be the seventh partial of 13/8. In this context, however, both ratios should be considered as part of the seventh-partial harmonic structure. Johnston uses these common-tone relationships in his music for modulation.

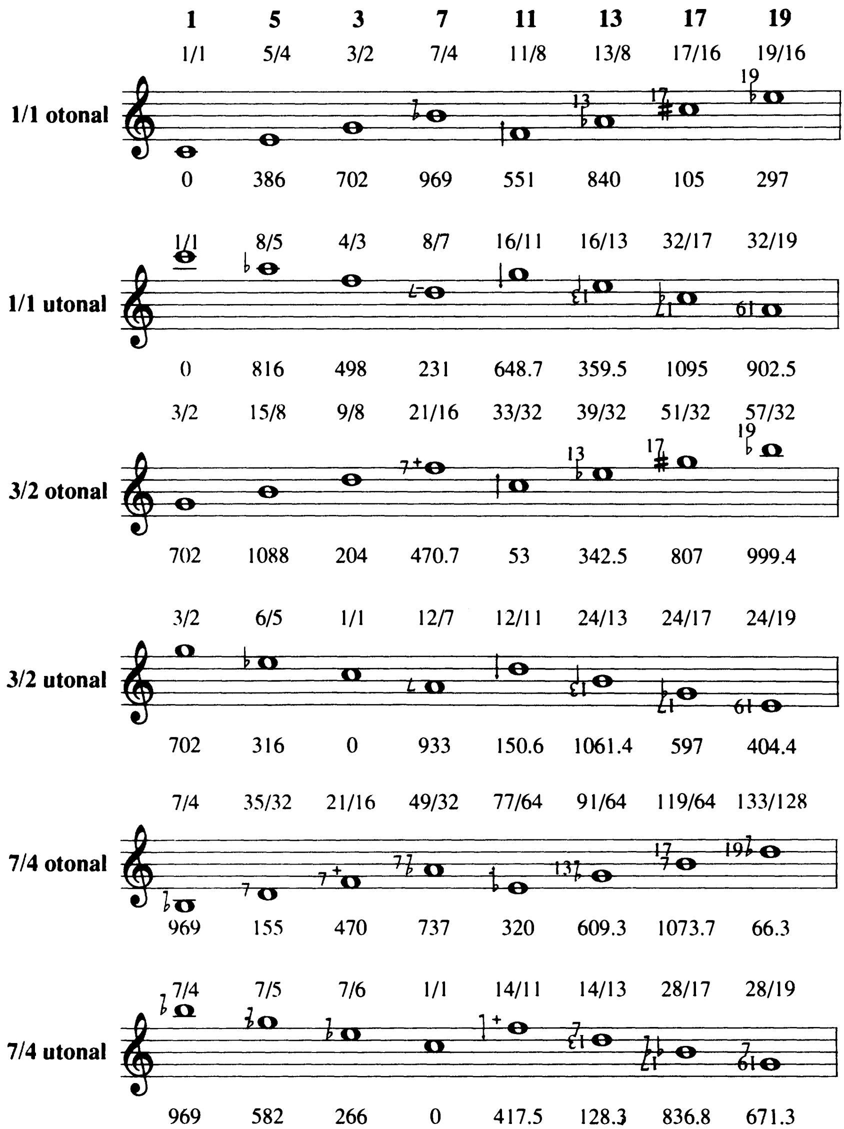
Example 17 presents the notation of the second, third, fifth, seventh, eleventh, thirteenth, seventeenth, and nineteenth partials of 1/1, 3/2, and 7/4. The ratios are given above, and the cent values below, the corresponding note in staff notation. It should be mentioned that, to date, Johnston's music has not used partials higher than the thirteenth in combination, even though some are represented in this example. The pitches generated in these and other harmonic structures are potential members of numerous types of both harmonic and scalar formations, but it would take considerable effort to methodically describe all those extensive possibilities. Certainly that task is the next step and it might illuminate the limits of human capability to perceive as well as reproduce such an enormous amount of discrete pitch information.

Before taking that step it is imperative to study Johnston's music, searching for his methods of pitch organization. Such a study should reveal potentials for new harmonic structures used in and implied by his music.

II. Music Examples

The music examples and brief analyses below are devoted to several of Johnston's string quartets which use the prime numbers 7 and higher. This article does not include String Quartet Number 4 since it has been discussed thoroughly by Randall Shinn (1977). These analyses provide information on certain types of harmonic and melodic structures found in much of Johnston's music. A thorough and coherent theory on the types of new harmonic potentials has not been developed at this time, although Johnston's music points to possible systems. These will be discussed. It is not within the scope of this article to offer extensive analyses, but rather to show particular excerpts that should imply further theoretical descriptions. Heidi Von Gunden (1986) makes some brief statements about String Quartets 5, 6, and 7, and there are a few necessary overlaps between this paper and her book.

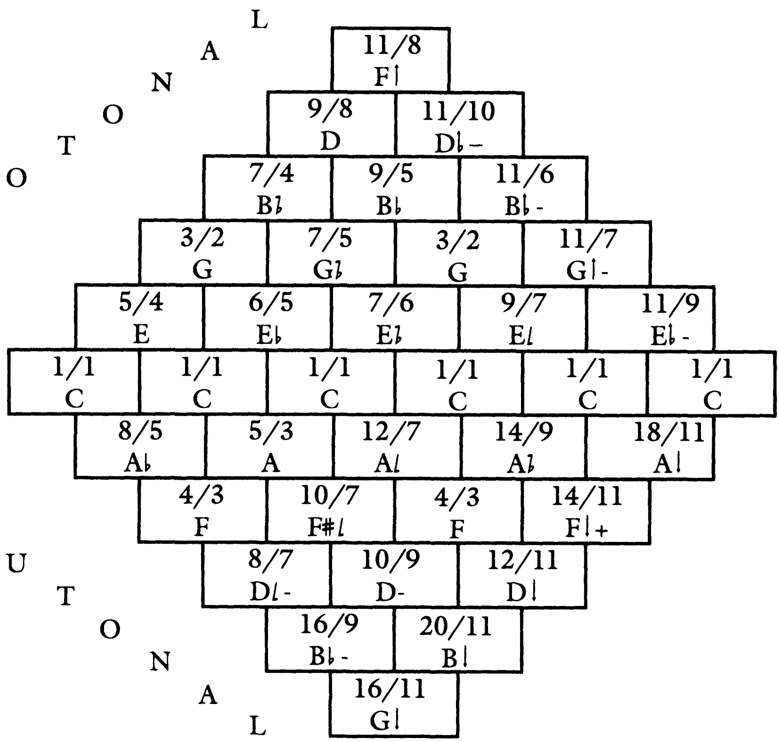
In general terms, Johnston's music eludes easy categorization. His stylistic and compositional methods run a broad gamut, from the topologically simple and yet complex String Quartet Number 4, the relentlessly rational String Quartet Number 6, the exceedingly complex String Quartet Number 7, to the joyous but defiant String Quartet Number 9. He is slave of no particular system other than extended just intonation; he uses serial processes, folk-like idioms, repetitive processes, traditional forms like fugue and variations, and



EXAMPLE 17: NOTATION OF PARTIALS 1, 3, 5, 7, 11, 13, 17, AND 19 FOR 1/1, 3/2, AND 7/4

intuitive processes, depending on his musical intentions. Nonetheless, in all these styles and processes certain harmonic and scalar structures do consistently emerge.

The harmonic structures are most logically described in terms of a chordal formation derived and extended from Harry Partch, with whom Johnston worked for a time. Partch's percussion instrument, the diamond marimba, is constructed to articulate this structure with ease. The six full otonal and six full utonal eleventh chords are based on the 2,3,5,7, and 11 limit. The formation is 1/1, 5/4, 3/2, 7/4, 9/8, 11/8 for otonal, and 1/1, 8/5, 4/3, 8/7, 16/9, 16/11 for utonal. These chords are projected on this instrument in a diamond shape, shown in Example 18.



EXAMPLE 18: PARTCH'S DIAMOND-MARIMBA PITCH BLOCK

Whereas Partch chose to build instruments to solve certain obvious problems and consequently forced a closed system, Johnston has quite willingly chosen to ignore instrument and performer limitations by not imposing arbitrary limits to harmonic and scalar potentials. Thus, his system allows for modulation and transposition to numerous pitch centers in keeping with his goal of extending western art music, something in which Partch was not interested at all. Johnston's music has also moved gradually and consistently upward in the harmonic series beyond the eleven limit and now includes the thirty-first partial.

In regard to harmonic organization, Johnston has extended Partch's eleventh-chord idea to a full thirteenth chord, and uses the higher partials as significant alternative chroma within that structure. The implications of this approach are enormous. Take for example the collections of significant intervals in Examples 19a and 19b, which present only a few of the most obvious potentials. The numbers at the top represent the chord member and the ratios in the boxes are possible chord tones. In this example the 1/1 and 3/2 remain immutable, in order to form a solid harmonic grounding.

chord members:	1	3	5	7	9	11	13	partials:
	1/1	5/4 6/5	3/2	15/8 16/9 9/5	9/8 16/15 27/25	4/3 45/32 27/20	5/3 27/16 8/5	2nd,3rd 5th
		16/13		7/4		11/8 16/11	13/8	7th,11th 13th
		19/16		32/17 29/16 31/16	17/16 32/31 32/29	23/16 32/23	32/19	17th,19th 23rd,29th 31st

EXAMPLE 19A: COLLECTION OF SIGNIFICANT PRIMARY INTERVALS

chord members:	1	3	5	7	9	11	13	partials
		7/6 9/7		27/14	21/20 15/14	7/5 10/7 21/16	14/9 12/7	7th
		11/9 27/22		11/6 20/11	11/10 12/11	16/11	33/20 18/11	11th
		13/10		39/20 24/13	13/12 40/39	13/9 18/13	20/13	13th

EXAMPLE 19B: COLLECTION OF SIGNIFICANT HIGHER-PARTIAL INTERVALS

In Example 19a most of the ratios are generated from the harmonic series of 1/1, or its most closely related diatonic ratios. Those in Example 19b are from a higher partial's harmonic series or closely related diatonic collection. The combinational potential of these collections is mind-boggling. If one also assumes these collections can be transposed to any other pitch within the system, the dimensions become completely absurd. By necessity, Johnston's systems of organization have imposed limitations by means of using straightforward harmonic relationships within musical procedures and structures bounded by very strict rules.

Before examining the music examples, a thorny issue present in some of Johnston's music and raised in Examples 19a and 19b must be addressed. This issue centers around the problem of harmonic generation (otonal and utonal) and musical context. In Examples 19a and 19b both otonal and utonal ratios, and also higher-partial chroma not from 1/1's harmonic series are presented as significant altered chroma. As mentioned in Part I of this paper, some of Johnston's music addresses more traditional tonal/modal orientations. In these works the functioning "root" of a chord is not necessarily identical with the otonal or utonal generator. Other works, however, do treat these generators as "roots." Therefore, Johnston's harmonic structures run the entire range from simple to complex and from lucid to ambiguous.

In one of the more transparent works, String Quartet Number 6 ("The Unending Melody"), Johnston uses a single harmonic structure (or chord type) in both the otonal and utonal forms. This structure (Example 20) is based on the 2, 3, 5, 7, and 11 limit, and modulates to numerous pitch centers following a logic born of serial processes and common-tone relationships.

Otonal						
1	3	5	7	9	11	(13)
1/1	3/2	5/4	7/4	9/8	11/8	(27/16)
C	G	E	B7	D	F↑	A+
Utonal						
1/1	4/3	8/5	8/7	16/9	16/11	(32/27)
C	F	A♭	DL−	B♭−	G↓	E♭−

EXAMPLE 20: HARMONIC STRUCTURE IN STRING QUARTET NO. 6

The 27/16 and 32/27 in parentheses are the Pythagorean major sixth and minor third respectively. They do not always appear in the harmony but usually do, as the inversional relation between the two hexachords which form the row (Example 21) provides those notes in the opposite hexachord. The three interlocking perfect fifths 1/1, 3/2, 9/8, 27/16 (D−, A, E, B in the prime-form transposition of this row) give this sonority a particularly brilliant quality. The 1/1s of the two hexachords are separated by a 135/128 (that is, the second hexachord maps onto the first under inversion and transposition by 135/128) and thus this interval is heard throughout the work as the hexachordal harmonies shift from one pitch center to the next. Example 21 shows the two hexachords forming the row, and the row's inversion.

Prime

A	D–	G↑–	F#	C7	E	/	G#	D#	A#↑+	B	E#7	C#
3/2	1/1	11/8	5/4	7/4	9/8		4/3	1/1	16/11	8/5	8/7	16/9
		(D– otonal)							(D# utonal)			

Inversion

A	E	B↓	C	F#7	D–	/	B♭–	E♭–	A↓–	G–	DL	F
4/3	1/1	16/11	8/5	8/7	16/9		3/2	1/1	11/8	5/4	7/4	9/8
		(E utonal)							(E♭– otonal)			

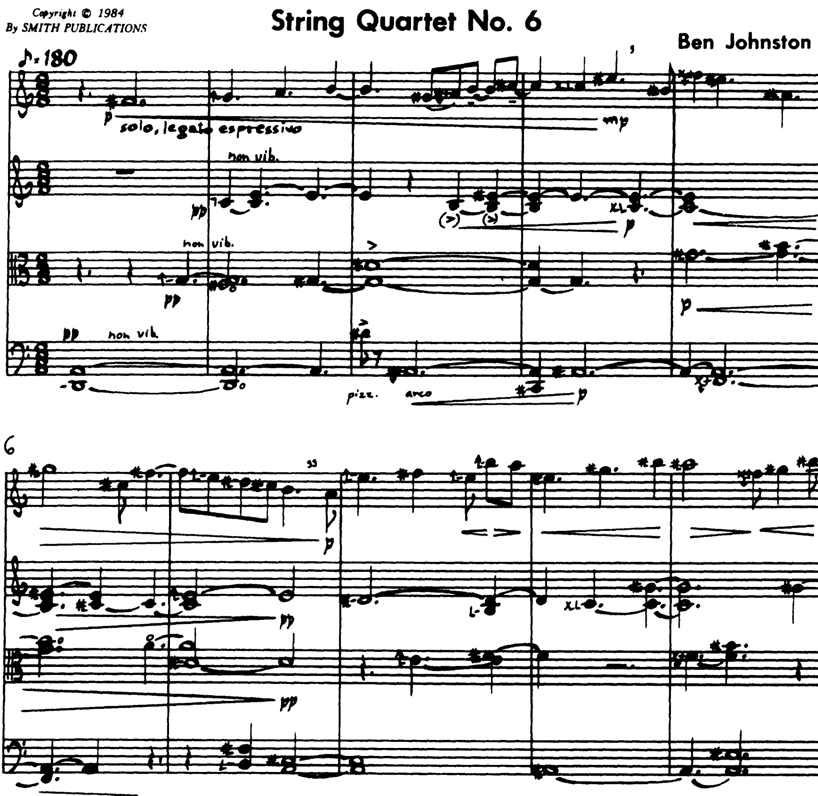
EXAMPLE 21: ROW FOR STRING QUARTET NO. 6

The classical row class produces a scale of sixty-three notes. It also produces twelve otonal and twelve utonal harmonic structures.[5] The choice of which row follows which is based on a common tone shared by the hexachords in the rows. The melodic writing is free but is taken from the harmonic structure. One instrument is given the melody while the other three provide harmonic support. The harmonic motion is quite slow, generally two measures for each chord. The whole composition is a large palindrome and there are numerous smaller palindromes and pseudo palindromes, as the following list of roots for the first six harmonic structures will show: D– o, D# u; B# u, B↓ – o; D# u, and D– o (the small o and u indicate otonal and utonal respectively). Example 22 gives the first ten measures of String Quartet Number 6.

String Quartet Number 7's second movement, entitled "Palindromes," uses a similar harmonic construction and compositional process. The "thirteenth" of the chord, however, is taken from the thirteenth partial instead of the 27/16, and it is always present. The ninth is produced by the same means as for the hexachords in String Quartet Number 6; however, it too is always present in the harmony of this quartet (Example 23).

The row and its inversion for the second movement of String Quartet Number 7 are given in Example 24. The two hexachords are related by retrograde inversion, at a transposition of interval 27/16 (C otonal down to A+ utonal). The classical row class produces twelve otonal and twelve utonal harmonic structures.

This movement also uses a single solo line supported by pizzicato chords



EXAMPLE 22: STRING QUARTET NO. 6, MM. 1–10

	1	3	5	7	9	11	13
Otonal:	1/1	3/2	5/4	7/4	(9/8)	11/8	13/8
Utonal:	1/1	4/3	8/5	8/7	(16/9)	16/11	16/13

EXAMPLE 23: HARMONIC STRUCTURE OF STRING QUARTET NO. 7, MOVEMENT 2

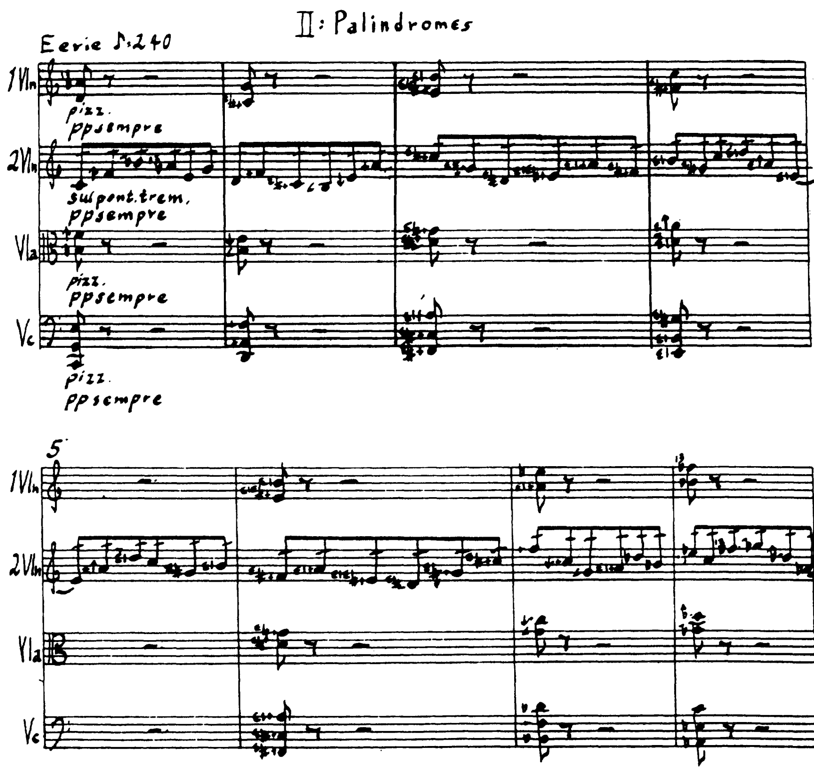
composed entirely of the notes in the harmonic structures. The first four measures (Example 25) contain the following progression of four hexachords: C o, A+ u; C#+ u, E o. The first two chords share a common note G; the next pair share C#+; and the last pair share B, in keeping with the common-tone progression technique used in String Quartet Number 6.

Prime

C	F↑	B7	$A^{13}\flat$	E	G	/	D	F+	$C^{ƐI}\sharp$	Bㄥ	E↓	A+
1/1	11/8	7/4	13/8	5/4	3/2		4/3	8/5	16/13	8/7	16/11	1/1
		(C otonal)							(A+ utonal)			

Inversion

C	G↓	Dㄥ–	$E^{ƐI}$	A♭	F	/	B♭–	G–	$C^{13}\flat$–	D7–	A↑♭–	E♭–
1/1	16/11	8/7	16/13	8/5	4/3		3/2	5/4	13/8	7/4	11/8	1/1
		(C utonal)							(E♭– otonal)			

EXAMPLE 24: ROW AND INVERSION FOR STRING QUARTET NO. 7, MOVEMENT 2

EXAMPLE 25: STRING QUARTET NO. 7, MOVEMENT 2, MM. 1–8

String Quartet Number 5 ("The Lonesome Valley" Quartet) uses no strict serial process, and thus the harmonic content and modulations are the result of a more intuitive logic. In the opening seven measures the theme is stated in the first violin while the others provide harmonic support with shifting upper-partial chroma. All the notes used in this opening, including the seventh and eleventh partials, are derived from a G– and D– utonal series, although the harmony is centered on C– minor[6] and the melody is grounded in F–. The scale formed by these two series (excluding the thirteenth for the present), constructed on a 1/1 of C–, is shown in Example 26.

C–	Dꜜ–	D–	E♭–	Eℓ–	F–	G–	Aꜜ	Aℓ–	B♭–
1/1	12/11	9/8	6/5	9/7	4/3	3/2	18/11	12/7	9/5

EXAMPLE 26: SCALE IN MM. 1–7 OF STRING QUARTET NO. 7

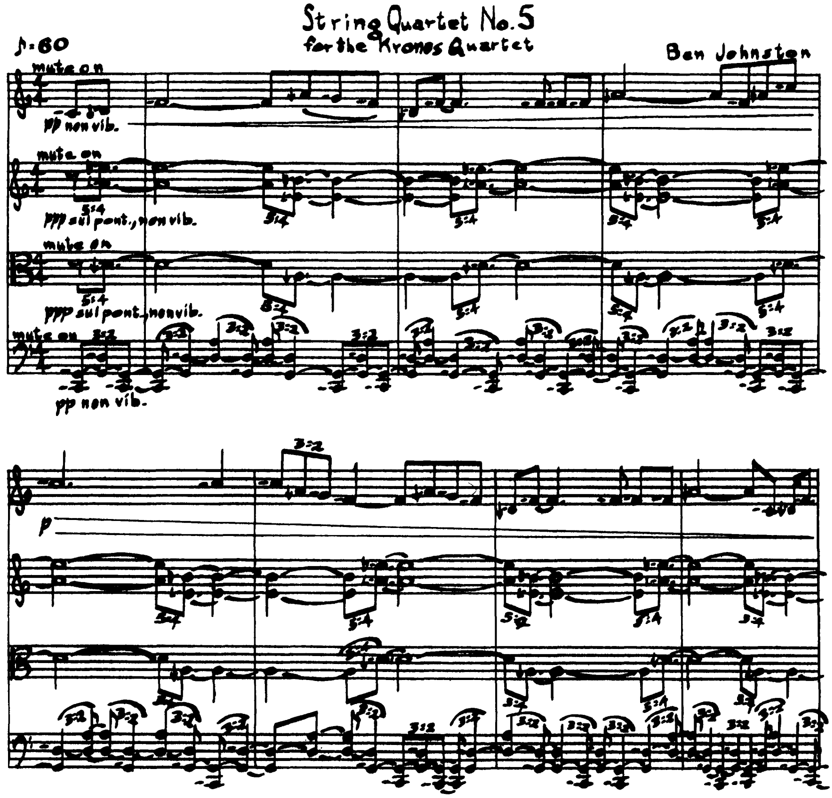
Since the opening of this work is centered on C–, the chords can be considered to transform as in Example 27.

1	3	5	7	9	11	13
C–	E♭–	G–		D–	F–	A
				Dꜜ–		Aℓ–
C–	E–	G–	B♭–	D–	F–	A
				Dꜜ–		Aꜜ

EXAMPLE 27: HARMONIC TRANSFORMATION IN MM. 1–7
OF STRING QUARTET NO. 5

A neutral third is found in the triad F– Aꜜ C–. The Aꜜ is a 27/22 from F– and the interval therefore measures 354.5 cents (compared to the just minor third's 318 and the just major third's 386). The 9/7 between C– and Eℓ– is an extremely large third having a cent value of 435: nearly 50 cents above the 5/4 (just major third). The seventh and eleventh partials are used to create a neutral third reminiscent of the "blue note" used in Sacred Harp or Gospel singing. Regardless of reference, Johnston uses the neutral third prominently in the opening theme.

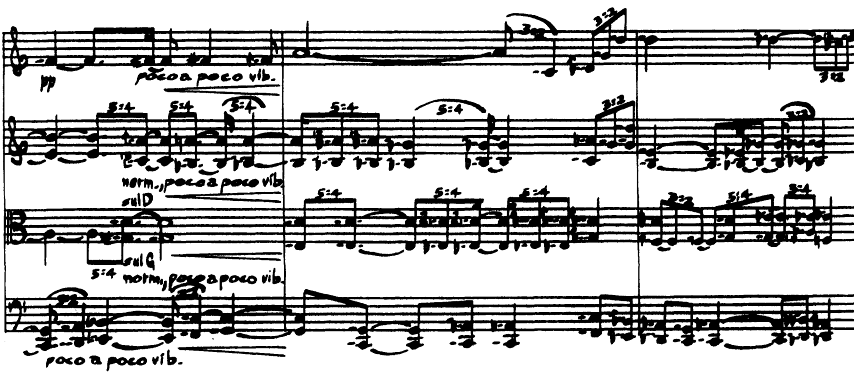
Although the theme is a simple folk song, the use of higher-partial chroma, especially between the members of the seventh and thirteenth tones, gives this music an extremely dissonant and unsettling poignancy (Example 28). With-



EXAMPLE 28: STRING QUARTET NO. 5, MM. 1–7

out a dedicated performance of these higher partial chroma, the music is completely trivialized.

This work uses the thirteen limit, and thus a full thirteenth chord formed on C– would be C–, E–, G–, B7–, D–, F↑–, and A^{13}♭ –. Measure 8 (Example 29) begins a section of harmonic motion which introduces the thirteenth partial. The harmonic motion is described in the following analysis. On the first beat of measure eight, while part of the F– utonal chord still holds, there is a passing use of A– utonal. Both of these are suspended on beat two, where a complete E– otonal chord resolves them. On the third beat this gives way to F– otonality which is overlaid by G– utonal plus D– otonal. On the fourth beat, C– otonality enters and lasts through beat one of measure 9. On beat two, this changes to B7– utonality (7/4 to C–), and on beat three to A♭ – otonality. On beat four, F– utonality is combined with G♭ – – otonality, to

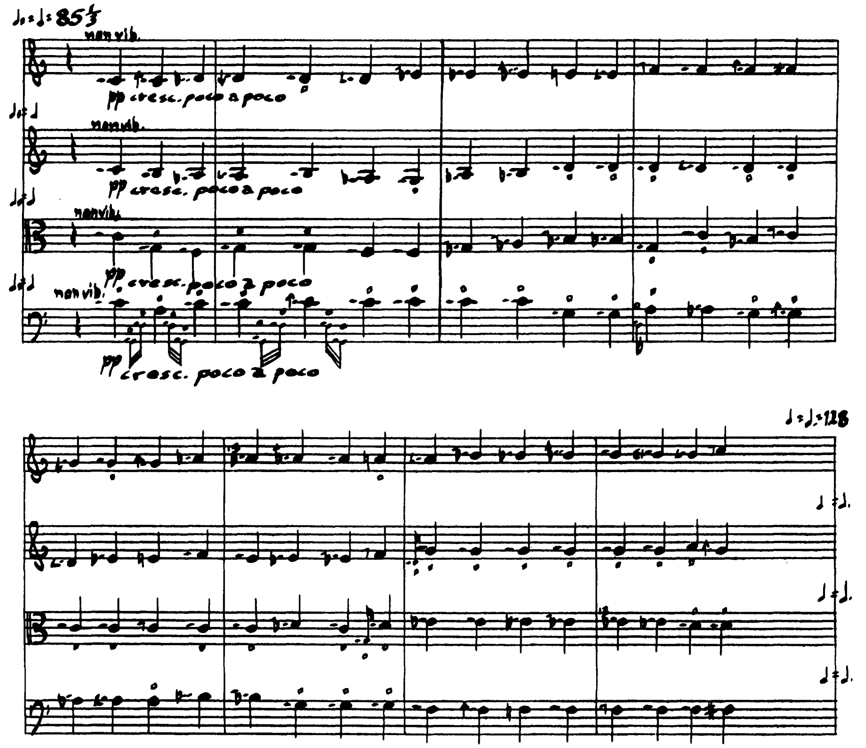


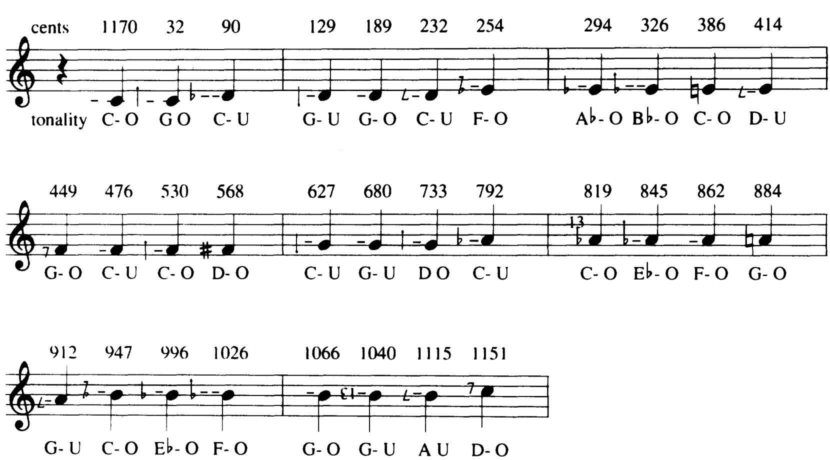
EXAMPLE 29: STRING QUARTET NO. 5, MM. 8–10

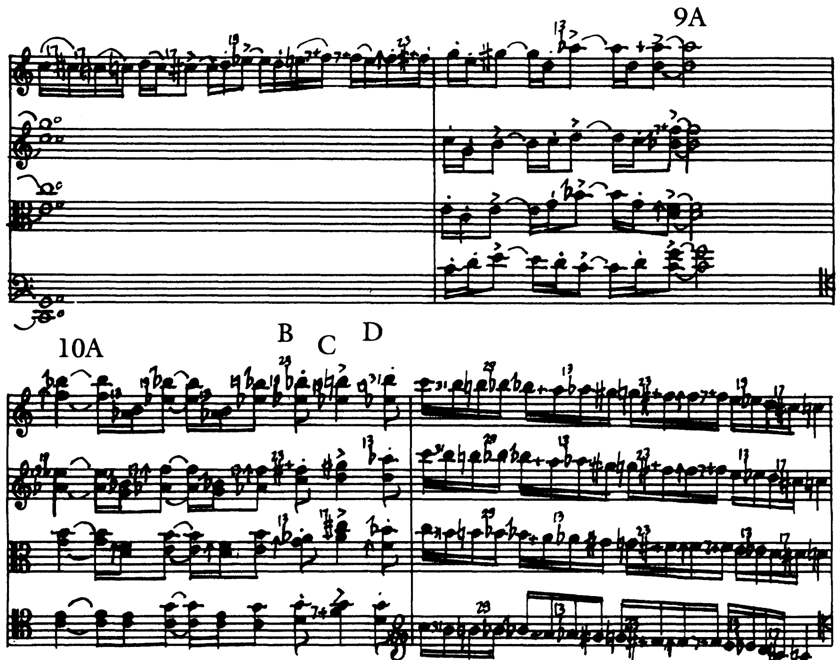
which is added an E♭ – – and a C– otonality. This last is held over into the first beat of measure 10 and is combined with D♭ – – otonality (11/8 of A♭ –), which resolves to C– otonality. On beat two, A♭ – otonality intervenes, followed on beat three by D7 – utonality (7/4 of E♭ –). On the fourth beat, E♭ – otonality occurs.

Measures 186–93 (Example 30) seem to summarize much of the harmonic and melodic material in a reflective moment near the end of the piece, just after the 'cello and then the second violin once again repeat the opening theme, slightly transformed. In this example it is clear that these harmonies are centered around C– minor (G– utonal), and function much like modal/tonal harmonic progressions.

The scale in the first violin (Example 31) is revealing in that it progresses upwards by small increments and at the same time is associated with simple modal harmonies. In Example 31, the cent values are given above each note of this melody, and the accompanying chord's tonality is given below.

The last movement of String Quartet Number 9 uses all the prime-number partials up to and including 31. Although this is the first time Johnston has used the twenty-third, twenty-ninth, and thirty-first partials, he does so in obvious harmonic and melodic ways and never strays far from C or G. In measures 9 and 10 (Example 32), the harmonic transformations appear to be the result of microtonal voice leading in the two violins. The same premise as used for the other quartets, however, can be used here to consider the harmonic structure in terms of some type of full thirteenth chord. The chord in measure 9 (marked 9A) is identical to those found in the Sixth and Seventh Quartets, except that the chord eleventh (*not* the eleventh partial) now includes a seventh-partial ratio, a 3/2 above the 7/4, which is 21/16 (F7 +). This is 80.6 cents lower than the eleventh *partial*, 11/8 (F↑).

EXAMPLE 30: STRING QUARTET NO. 5, MM. 186–93



EXAMPLE 31: FIRST-VIOLIN MELODY AND HARMONIC PROGRESSION IN MM. 186–93 OF STRING QUARTET NO. 5

EXAMPLE 32: STRING QUARTET NO. 9, MOVEMENT 4, MM. 9–11

In Example 33 one can observe the transformation of harmonic structures by the changing of higher-partial chroma. The C disappears after the first three chords, while the G is retained throughout and the D is absent only from chord 10A. The third (E) is absent from chord 9A, perhaps because of the two competing elevenths (Fs). Five of the chord members have two notes in them, resulting in even more tension. The absence of C in the chords 10C and 10D might bring into question this heptachordal, thirteenth-chord approach as an explanation of the harmonic structure. It does, however, provide a basis for tuning, so necessary for an accurate performance of Johnston's music. To stretch this notion even further, a case could be made for a dominant function of chords 10C and 10D. Three facts support an interpretation of these chords as dominant structures on G (Example 34): C is absent, the music is lunging toward C (marked by an abrupt, one-octave descending unison scale in measure 11), and the four chord members G, B7 (B^{31} in chord 10D), D, and F7 + form a strong dominant-seventh sonority.

The descending scale in measure 11 of the Ninth Quartet (Example 32) is composed of notes all appearing in the preceding measures, and are the sixteenth through thirty-second partials of C. The scale has the following cent values, descending from C: 1200, 1145, 1088, 1029, 969, 906, 840, 772,

chord member:	1	3	5	7	9	11	13	Measure
	C	(E)	G	B7	D	F7 + F↑	A+	9A
	C	E E♭^{19}	G	B7 B		F↑	A♭^{13}	10A
	C	E♭^{19}	G	B♭^{29}	D	F#+^{23} F↑	A♭^{13}	10B
		E♭^{19}	G	B	D C#^{17}	F7 +	A+ G#	10C
		E♭^{19}	G	B^{31} B7	D	F↑	A♭^{13}	10D

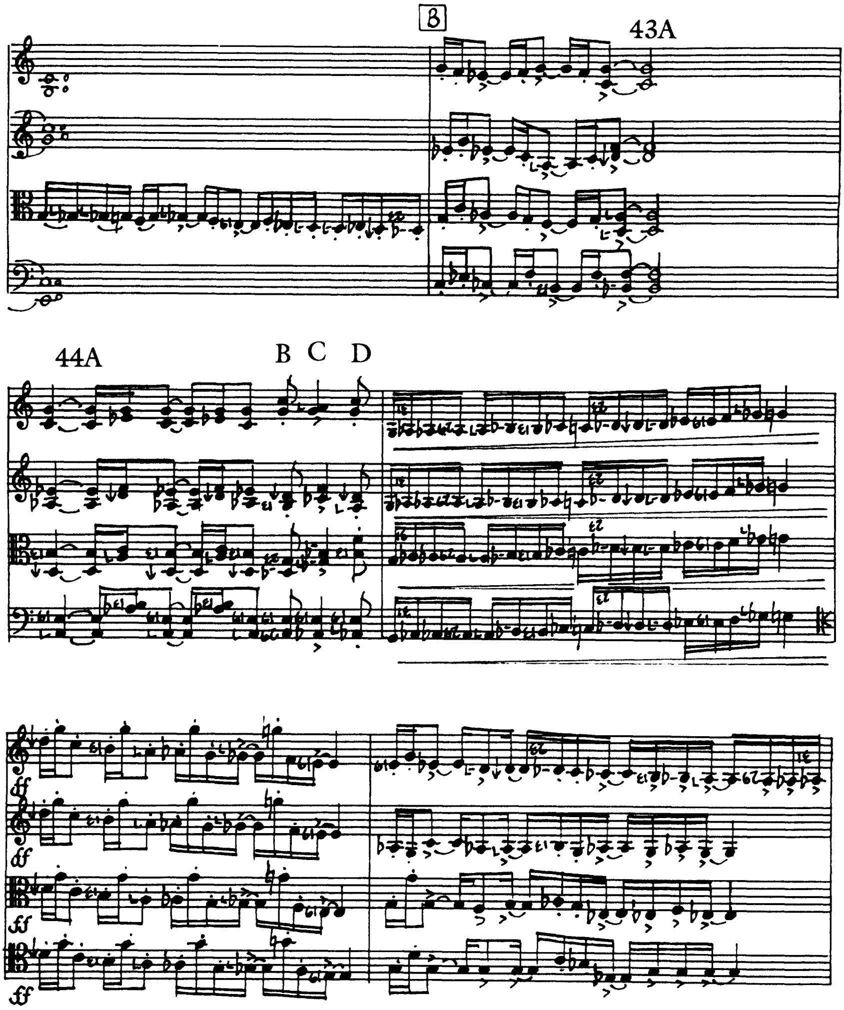
EXAMPLE 33: HARMONIC TRANSFORMATIONS IN STRING QUARTET NO. 9, MOVEMENT 3, MM. 9–10

1	3	5	7	9	11	13
G	B^{31} (B7)	D	F↑ F7 +	A♭^{13}	C#^{17}	E♭^{19}

EXAMPLE 34

702, 628, 551, 471, 386, 296, 208, 105, 0. The descending intervals get progressively larger (55, 57, . . . , 103, 105) until the last is almost double the size of the first.

A construction similar to that of Example 32 is found in measures 43–46 (Example 35), which is almost an exact inversion of Example 32, formed on G utonal. There are some differences, perhaps the most curious being the A♭L rather than A♭^{ıɛ} for the 'cello's last note in measure 45.

Although this thirteenth-chord approach might be distorting the actual compositional process Johnston used in these passages (Example 33 and 36), it might also help the interpreter establish some guidelines for interval relationships and tuning. Furthermore, this approach might provide a foundation toward a theory of new harmonic potentials offered by the abundant resources of extended just intonation.

EXAMPLE 35: STRING QUARTET NO. 9, MM. 43–46.

1	3	5	7	9	11	13	Measure
G	(E♭)	C	A$\rotatebox{180}{7}$	F	D$\rotatebox{180}{7}$– D↓	B♭–	43A
G	E♭ E$^{\rotatebox{180}{19}}$	C	A$\rotatebox{180}{7}$ A♭		D↓	B	44A
G	E$^{\rotatebox{180}{19}}$	C	A$^{\rotatebox{180}{29}}$		D D♭$^{\rotatebox{180}{23}}$–	B$^{\rotatebox{180}{13}}$	44B
G	E$^{\rotatebox{180}{19}}$	C♭	A$\rotatebox{180}{7}$ A♭	F G$^{\rotatebox{180}{17}}$♭–		B♭–	44C
G	E$^{\rotatebox{180}{19}}$	C	A$\rotatebox{180}{7}$ A♭$\rotatebox{180}{7}$	F	D↓	B$^{\rotatebox{180}{13}}$	44D

EXAMPLE 36: HARMONIC TRANSFORMATIONS IN STRING QUARTET NO. 9, MOVEMENT 4, MM. 43–46

NOTES

1. Ben Johnston "On the Performance Practice of Extended Just Intonation," from "Notes" to the score of String Quartet Number 9.

2. All of these works are published by Smith Publications, Baltimore, Maryland.

3. For instance, most of his works utilize A = 440 as the tuning note. This A is a 5/3 to C, therefore the tuning of C is down a just 5/3 from A and is 264 Hertz rather than the twelve-equal-tempered (261.626), or about 16 cents higher than the tempered. Also, in this article all cent values are approximate, and all compound intervals have been reduced so as to lie in the octave between 1/1 and 2/1.

4. A complete listing (notation, ratios, and cent values) of all the lattice structures up to 31, including all possible combinations Johnston has used, is available from the author.

5. For a more detailed discussion, see Steven Elster, "A Harmonic and Serial Analysis of Ben Johnston's String Quartet No. 6," *Perspectives of New Music* 29, no. 2 this issue.

6. The tuning of the open strings for all the strings is C–, G–, D–, A (440), E. This is not "scordatura." Therefore, the natural resonance of the instruments is exploited in C–.

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Scalar Order as a Compositional Resource
Ben Johnston
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